CSE 140: Components and Design Techniques for Digital Systems

Lecture 6: Universal Gates

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Combinational Logic: Various Types of Gates

- Universal Set of Gates
  - Motivation
  - Definition
  - Examples
- Other Types of Gates
  - XOR
  - NAND / NOR
  - Block Diagram Transfers

Universal Set of Gates: Motivation

- AND, OR, NOT: Logic gates related to reasoning from Aristotle (384-322BCE).
- NAND, NOR: Inverted AND, Inverted OR gates. For VLSI technologies, all gates are inverted (AND,OR operation with a bubble at output).
- XOR: Exclusive OR gates. Parity check.
- Multiplexer + input table: Table based logic for programmability. FPGA technology.
- Neuron and Synapse: Neural network.
- Reversible Gates: Quantum computing.

In the future, we may have new sets of gates due to new technologies. Given a set of gates, can the gates in the set cover all possible switching functions?

Universal Set

Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology.

Criterion: If the set of gates can implement AND, OR, and NOT gates, the set is universal.

Universal Set Definition

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT}

{OR, NOT}

Universal Set: Examples

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT} OR can be implemented with AND & NOT gates: $a + b = (a'b')'$

{OR, NOT} AND can be implemented with OR & NOT gates: $ab = (a' + b')'$

{XOR} is not universal

{XOR, AND} is universal

iClicker

Is the set {AND, OR} (but no NOT gate) universal?

A. Yes

B. No

Note that the set was used in a once popular design style as domino logic for high performance computing.

iClicker

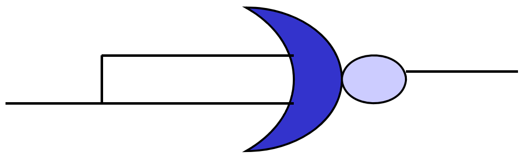
Is the set $\{f(x, y) = xy'\}$ universal?

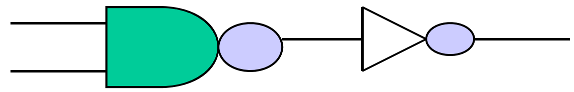
A. Yes

B. No

Universal Set: Examples

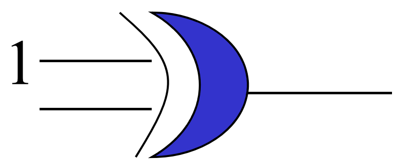
{NAND, NOR}





{XOR}

{XOR, AND}



$x \oplus 1 = x1' + x'1 = x'$ if constant "1" is available.

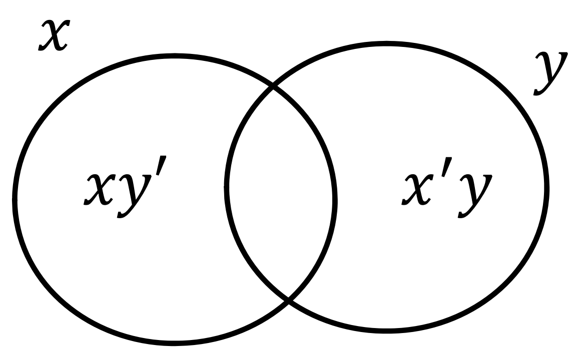
Other Types of Gates: Properties and Usage

1. XOR $x \oplus y = xy' + x'y$
2. NAND, NOR
3. Block Diagram Transfers

Other Types of Gates: XOR

$x \oplus y = xy' + x'y$

It is a parity function (examples) useful for testing because the flipping of a single input changes the output.

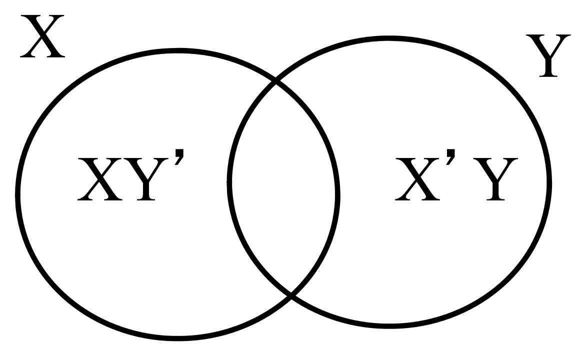


id	x	y	$x \oplus y$
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

	x=0	x=1
y=0	0	1
y=1	1	0

Other Types of Gates: XOR

1) XOR $x \oplus y = xy' + x'y$

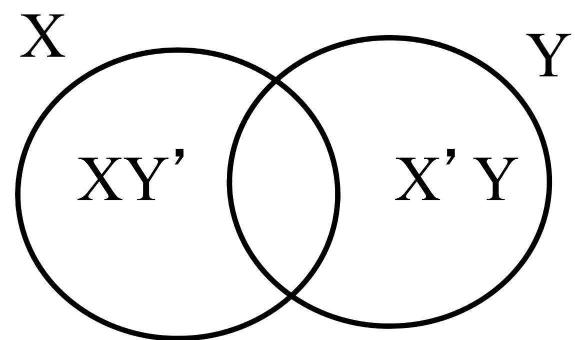


(a) Commutative $x \oplus y = y \oplus x$

(b) Associative $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

Other Types of Gates: XOR

1) XOR $x \oplus y = xy' + x'y$



a) Commutative $x \oplus y = y \oplus x$
b) Associative $(x \oplus y) \oplus z = x \oplus (y \oplus z)$
c) $1 \oplus x = x', 0 \oplus x = x$
d) $x \oplus x = 0, x \oplus x' = 1$

Other Types of Gates: Properties and Usage

e) If $ab = 0$, then $a \oplus b = a + b$

Proof: If $ab = 0$, then

$$a = a(b+b') = ab+ab' = ab'$$

$$b = b(a + a') = ba + ba' = a'b$$

Therefore, $a+b = ab' + a'b = a \oplus b$

Note that in full adder, we have

$$c_{out}=ab+bc+ac=ab+c(a+b)$$

From property e), we can also write $c_{out}=ab+c(a \oplus b)$

Other Types of Gates: XOR

f) $f(x, y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x = ?$

(Priority of operations: AND, $\oplus$, OR)
Hint: We apply Shannon's Expansion.

Shannon's Expansion (for switching functions)

Formula: $\boldsymbol{f(x,y) = xf(1,y) + x'f(0,y)}$

Proof by enumeration:
If $x = 1, f(x,y) = f(1,y)$:

$\boldsymbol{xf(1,y) + x'f(0,y)}$=$1f(1,y) + 1'f(0,y)$=$f(1,y)$

If $x = 0, f(x,y) = f(0,y)$:

$\boldsymbol{xf(1,y) + x'f(0,y)}$= $0f(x,y) + 0'f(0,y) = f(0,y)$

Other types of gates: XOR

Simplify the function (Priority of operations: AND, $\oplus$, OR)

$f(X,Y) = X \oplus XY' \oplus X'Y \oplus (X+Y) \oplus X$

Case X = 1: $f(1, Y) = 1 \oplus Y' \oplus 0 \oplus 1 \oplus 1 = Y$

Case X = 0: $f(0, Y) = 0 \oplus 0 \oplus Y \oplus Y \oplus 0 = 0$

Thus, using Shannon's expansion, we have

$f(X, Y) = Xf(1,Y)+X'f(0,Y)= XY$

XOR gates

iClicker: Is the equation

$$a+(b\oplus c) = (a+b)\oplus(a+c) \text{ true?}$$

A. Yes

B. No

Other Types of Gates: NAND, NOR

2) NAND, NOR gates

NAND (NOR) gates are not associative

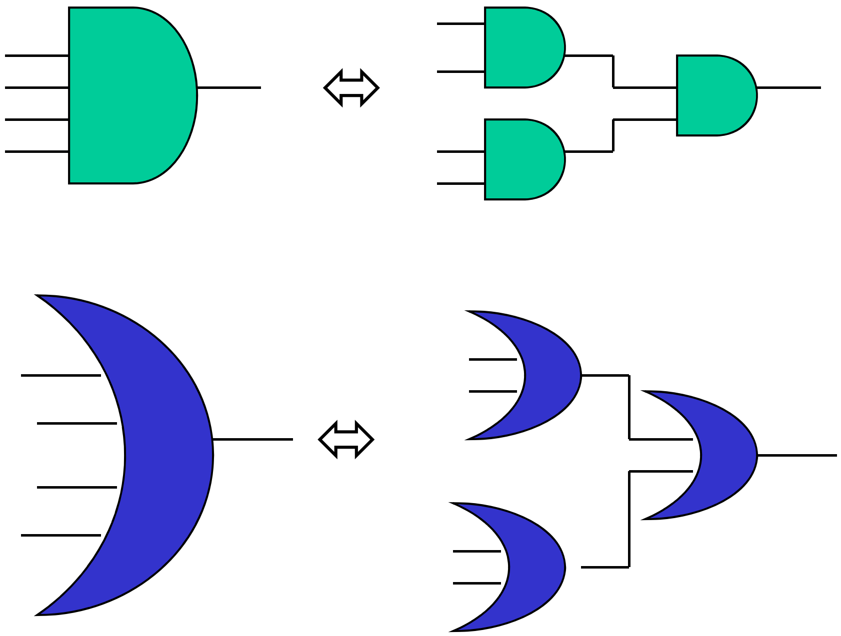
Let $a \mid b = (ab)'$

$$(a \mid b) \mid c \neq a \mid (b \mid c)$$

Other Types of Gates: Block Diagram Transform

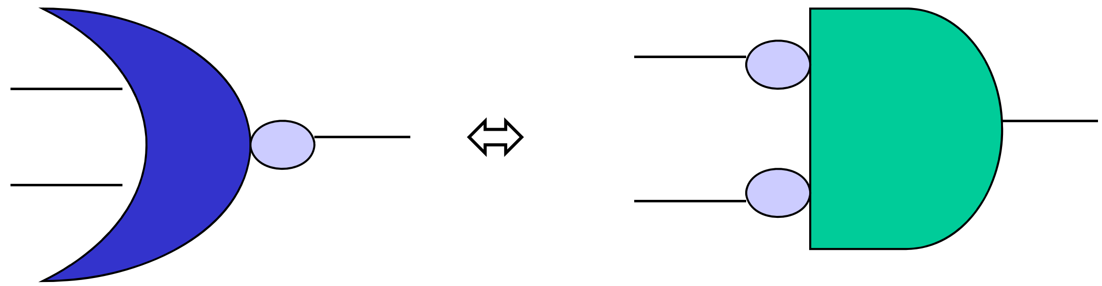
3) Block Diagram Transformation

a) Reduce # of inputs.

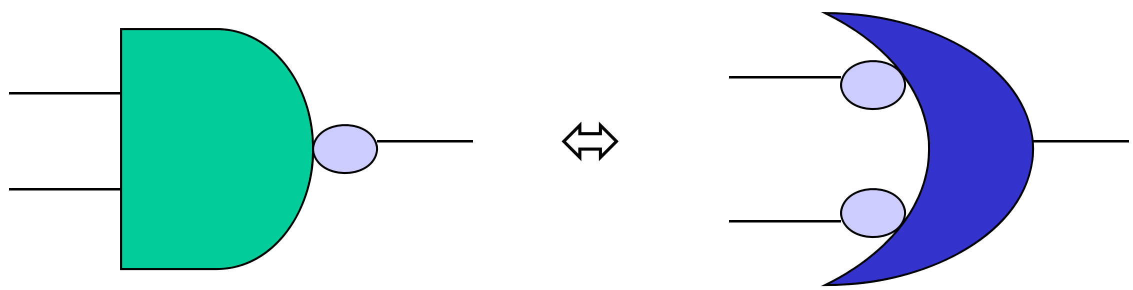


Other Types of Gates: Block Diagram Transform

b. DeMorgan's Law



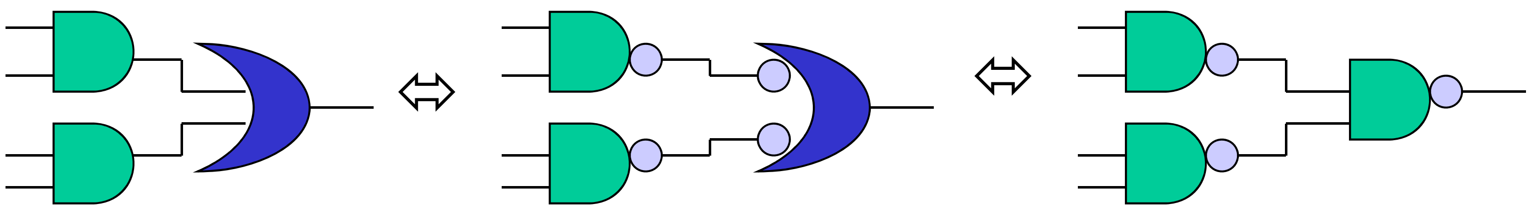
(a+b)' = a' b'



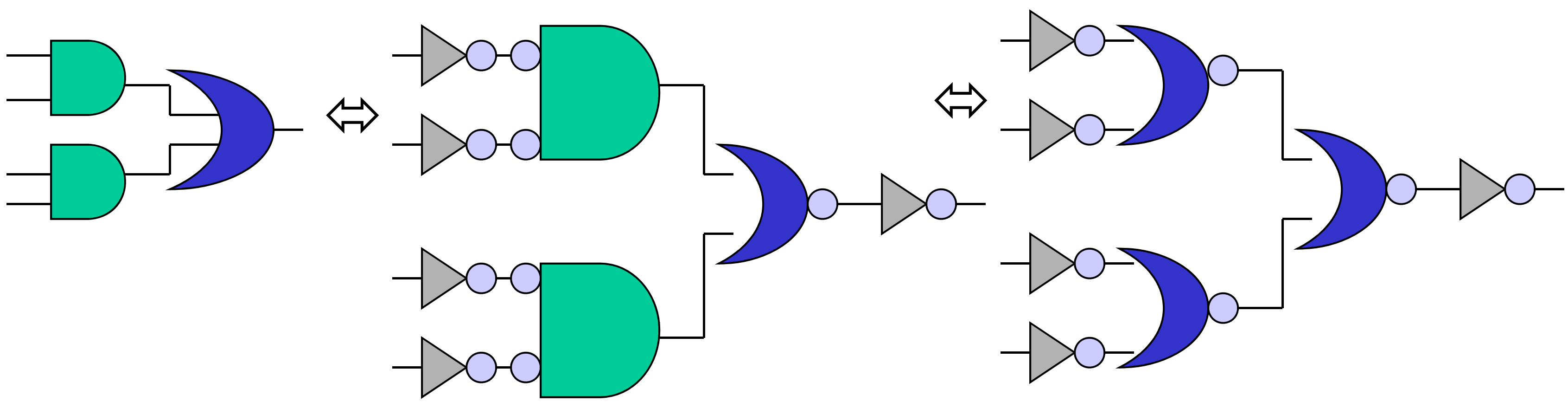
(ab)' = a' +b'

Other Types of Gates: Block Diagram Transform

c. Sum of Products (Using only NAND gates)

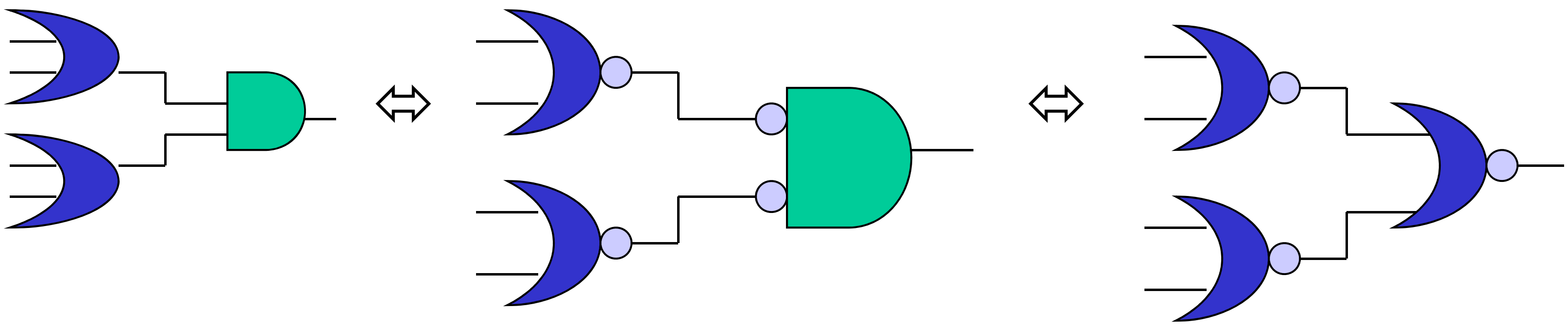


Sum of Products (We create many bubbles with NOR gates)



Other Types of Gates: Block Diagram Transform

d. Product of Sums (NOR gates only)



We will create many bubbles with NAND gates.

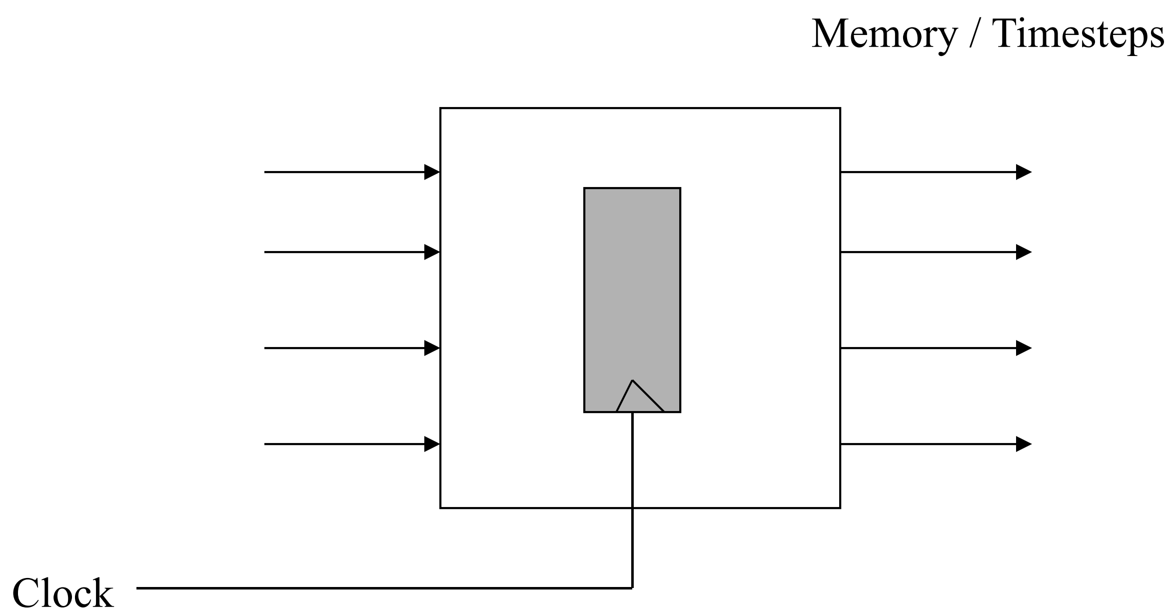
NAND, NOR gates

Remark:

Two level NAND gates: Sum of Products

Two level NOR gates: Product of Sums

Part II. Sequential Networks



Flip flops
Specification
Implementation